VERIFICATION

UDM CHAPTER 3: JET DISPERSION

DATE: December 2023

Reference to part of this report which may lead to misinterpretation is not permissible.

No.	Date	Reason for Issue	Prepared by	Verified by	Approved by
1	1999	PHAST 6.0	Witlox and Holt		
2	Oct 2005	SAFETI 6.5	Witlox and Harper		
3	May 2011	Phast 6.7, UDM v2	Harper		
4	Dec 2020	Phast 8.4	Harper	Hart	
5	May 2021	Apply new template	D. Vatier		

Date: December 2023

Prepared by: Digital Solutions at DNV

ABSTRACT

The UDM theory and solution algorithm for elevated dispersion and ground-level jet dispersion have been investigated in detail.

The UDM results are shown to be identical to the results obtained by an analytical solution for an elevated horizontal jet. Very good agreement has been obtained against the Pratte and Baines correlation for plume rise (no ambient turbulence). Improved predictions are shown against the Briggs correlation (including ambient turbulence).

Finally a sensitivity analysis has been carried out for a given base-case problem (jet dispersion of ‘air’). Parameter variations have been carried out to the release height, release speed, release angle and transition criterion.

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3 JET DISPERSION

3.1 Introduction

This chapter documents the verification and sensitivity analysis of the Unified Dispersion Model (UDM) for the case of continuous isothermal jet dispersion. It accompanies the theoretical description of the UDM[i].

In Section 3.2 an analytical solution of the UDM dispersion equations is derived in case of an elevated horizontal jet (jet entrainment only; no crosswind and passive entrainment, no airborne drag). The UDM numerical results are shown to be in exact agreement with the analytical results.

In Section 3.3 the UDM jet/cross-wind entrainment formulation and airborne drag (no passive entrainment) are verified against empirical correlations. The UDM is also compared against an empirical correlation for a buoyant plume. Finally UDM results have been compared against Briggs plume-rise correlations (including passive entrainment).

In Section 3.4 a sensitivity analysis has been carried out. Parameter variations have been carried out to the release height, release speed, release angle and transition criterion.

3.2 Horizontal jet (jet-entrainment)

In Section 3.2.1 an analytical solution is derived for horizontal elevated jets. This generalises the analytical solution by Long (quoted by Section 15.20.2 in Lees[ii]), and Webber and Kukkonen[iii] to non-zero wind speed. In Section 3.2.1 the analytical solution is obtained from a set of reduced equations directly obtained from the UDM algorithm. The solution is obtained for both cases of the Spalding jet-entrainment formulation and the Morton jet-entrainment formulation (see UDM theory manual for details). For zero wind speed the analytical solution is shown to reduce to the analytical solution provided by the above authors.

In Section 3.2.2 a horizontal jet released with 50 m/s speed at 50 m height is considered. The UDM numerical results are shown to exactly match the analytical results. Thus the UDM has been verified for the case of jet entrainment (no airborne or ground drag; no cross-wind or passive entrainment). Note that the jet entrainment coefficient α_1 has been chosen such that agreement is obtained between the maximum concentrations observed in the Spalding's experiments [see UDM theory manual for details].

3.2.1 UDM algorithm; analytical solution

UDM equations
The UDM theory manual includes a complete set of dispersion equations. For an horizontal isothermal, continuous, jet dispersion of 'air' these equations simplify as follows:

- zero water-vapour transfer from ground: $m_{wv}^{gnd}=0$
- no heat transfer from ground: $q_{gnd} = 0$
- enthalpy equation: $T_{cld} = T_a \rightarrow$ density $\rho=\rho_a$ $\rightarrow$ concentration profile m=n=2 [n>2 for non-neutral]
- constant horizontal excess momentum: $I_{x2} = m_{cld}(u_{cld}-u_a) = Q[u_{cld}(x=0)-u_a]$
- zero vertical momentum: $I_z = 0$, $u_z = 0$, z_{cld} = release height z_R , $\theta=0$
- cloud mass/area relation $m_{cld} = u_{cld}\rho_a A_{cld}(x)$
- cloud area $A_{cld}(x) = (1+h_d)\ \pi\ \sigma_y\sigma_z$
- cloud mass entrainment:
  $dm_{cld}/dx = Ent_{jet} = \alpha_1 m_{cld}\rho_{air}(u_{cld}-u_a)^{1/2}$ if jet,
  $dm_{cld}/dx = E_{pas} = A_{cld}(x)\ u_a\rho_a[\sigma_y^{-1}d\sigma_{ya}/dx+\sigma_z^{-1}d\sigma_{za}/dx]$ in far-field (transition in between)
- cloud spreading (see theory manual):
  * circular plume, if jet (see theory)
  * $dW_{eff}/dx = 0.5(2\pi)^{0.5}\ d\sigma_y/dx = 2^{0.5}\ C_m\, d\sigma_{ya}/dx(x-x_0)$, $C_m = 0.5\lambda\pi^{0.5}$, passive

where $Q = m_c$ is the release rate (kg/s).

Thus the following equations remain during the initial jet phase for u_{cld}, m_{cld}, σ_y, σ_z:

(a) $m_{cld}(u_{cld}-u_a) = I_{x2} = Q[u_{cld}(x=0)-u_a]$ (horizontal momentum)
(b) $dm_{cld}/dx = \alpha_1[m_{cld}\rho_a(u_{cld}-u_a)]^{1/2}$ or $dm_{cld}/dx=e_{jet}P_{above}\rho_a(u_{cld}-u_a)$ (jet entrainment)
(c) $m_{cld} = u_{cld}\rho_a A_{cld}(x) = u_{cld}\rho_a\ (1+h_d)\ \pi\ \sigma_y\sigma_z$ (cloud area)
(d) circular plume $\sigma_y = \sigma_z$

where for an elevated plume:

$$e_{jet}\ P_{above} = e_{jet}\ (2\ \pi R_z) = e_{jet}\ 2(\pi A_{cld})^{0.5} = \alpha_1\ A_{cld}^{0.5} = \alpha_1\ [m_{cld}/(u_{cld}\rho_a)]^{0.5}$$

Eliminating u_{cld} using Equation (a), Equation (b) can be expressed as a differential equation of $m_{cld}(x)$:

$$dm_{cld}/dx = \alpha_1\ [\rho_a\ I_{x2}]^{0.5} \quad \text{(Emerson formulation)}$$
$$= \alpha_1\ I_{x2}\ [\rho_a/(I_{x2} + u_a m_{cld})]^{0.5} \quad \text{(Morton et al. formulation)}$$

Analytical solution
Using Emerson formulation the above equation can easily be integrated, and it follows that

$$m_{cld}(x) = Q + \alpha_1\ [\rho_{air}\ I_{x2}]^{0.5}x$$
$$u_{cld}(x) = u_a + Q[u_{cld}(x=0)-u_a]\ /\ \{ Q + \alpha_1\ [\rho_a\ I_{x2}]^{0.5}x \}$$
$$A_{cld}\ (x) = m_{cld}/[u_{cld}\rho_a]$$

Using the formulation of Morton, Taylor and Turner, it follows that (different formula to be used for $u_a\approx0$!)

$$m_{cld}(x) = \frac{1}{u_a}\left\{-I_{x2} + \left[\left(I_{x2} + u_a Q\right)^{3/2} + 1.5\,\alpha_1 u_a I_{x2} \rho_a^{0.5}\,x\right]^{2/3}\right\}$$

$u_{cld}(x) = u_a + I_{x2}/m_{cld}$
$A_{cld}(x) = m_{cld}/[u_{cld}\rho_{air}]$

with the constant excess momentum $I_{x2} = Q[u_{cld}(x=0)-u_a]$.

Note that for both formulations for x=0, $m_{cld} = Q$, $u_{cld} = u_{cld}(x=0)$, $A_{cld} = Q/[u_{cld}(x=0)\rho_a]$. For $x \rightarrow \infty$, $u_{cld} = u_a$.

Further data are derived as follows:

- $c_o = Q / [u_{cld}A_{cld}(x)]$ = centre-line concentration
- $A_{cld}(x) = 2\pi\,\sigma_y\,\sigma_z$ [provided that $h >> \sigma_z$]
- $\sigma_y = \sigma_z$

Note that for $u_a = 0$, both formulations lead to identical results. For $u_a=0$, the expressions can be further simplified as

$m_{cld} = Q\,\{1 + (x/R_0)\tan(\beta_\infty)\}$, $u_{cld} = u_{cld}(x=0) / \{1 + (x/R_0)\tan(\beta_\infty)\}$,

$A_{cld}(x) = \pi R_o^2\{1 + (x/R_0)\tan(\beta_\infty)\}^2$, $R(x) = R_o\{1 + (x/R_0)\tan(\beta_\infty)\}$

where R_o is the initial jet diameter [$Q = \pi R_o^2\rho_{air}u_{cld}(x=0)$].

Note that the jet half-angle equals $\arctan\{[R(x)-R_o]/x\} = \beta_\infty$. Thus the definition of β_∞ is confirmed.

3.2.2 Basecase

Basecase input data

Basecase (as in passive but release velocity is 50 m/s, use 50 meter release height):

- case: continuous (duration 360000 s) 10 kg/s release of nitrogen_air [mat.no. = -1001] released at 50 m height with temperature 298K and **velocity 50 m/s**
- ambient: logarithmic-temperature and linear-pressure profile, D5 and 298K at 50 m (cut-off for wind = 1 m), solar flux = 500 W/m^2, air mole weight = 28.966 [possibly initially use constant profiles]
- substrate: dispersion over land – dry soil (temperature = 298K), surface roughness = 0.1m
- parameters: maximum distance = 100000, distance multiple for full passive entrainment = 2, dense to passive smoothing transition parameter = 2
- averaging time =600 s

Comparison of basecase against analytical solution (no passive transition)

First UDM runs of the basecase were carried out disabling the passive transition using both the Spalding and Morton's jet entrainment formulation. Exact agreement was obtained for cloud mass, cloud area, cloud radius (R_y=R_z), centre-line concentration and cloud velocity.

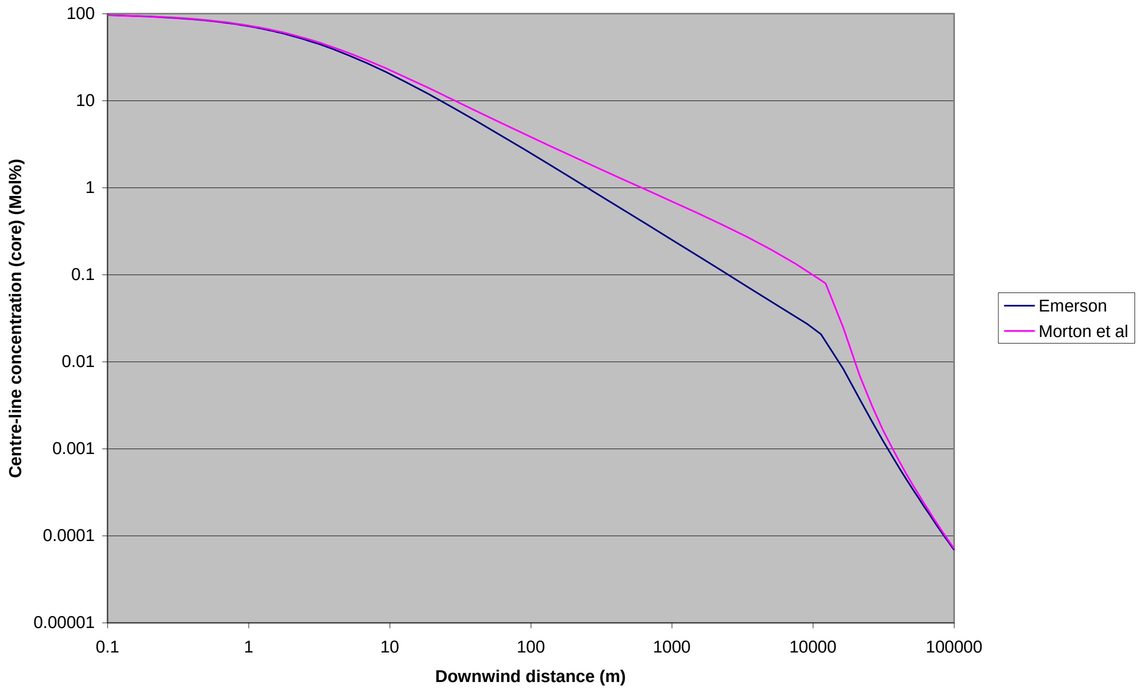
Figure 3.1 illustrates the differences on the dispersion results between the Emerson and Morton jet dispersion models. It include results for the centre-line concentration c_o, the cloud radius (R_y=R_z) and cloud cente-line velocity.

Comparing equations II and I it can be seen, that for a horizontal release whose density is the same as air, the ratio of entrainment rates can be given by the equation:

$$\frac{[E_{jet}]_{Spalding}}{[E_{jet}]_{Morton}} = \sqrt{\frac{\rho_{cld}}{\rho_a}\,\frac{u_{cld}}{u_{cld} - u_a\cos\theta}} = \sqrt{\frac{u_{cld}}{u_{cld} - u_a}} \geq 1$$

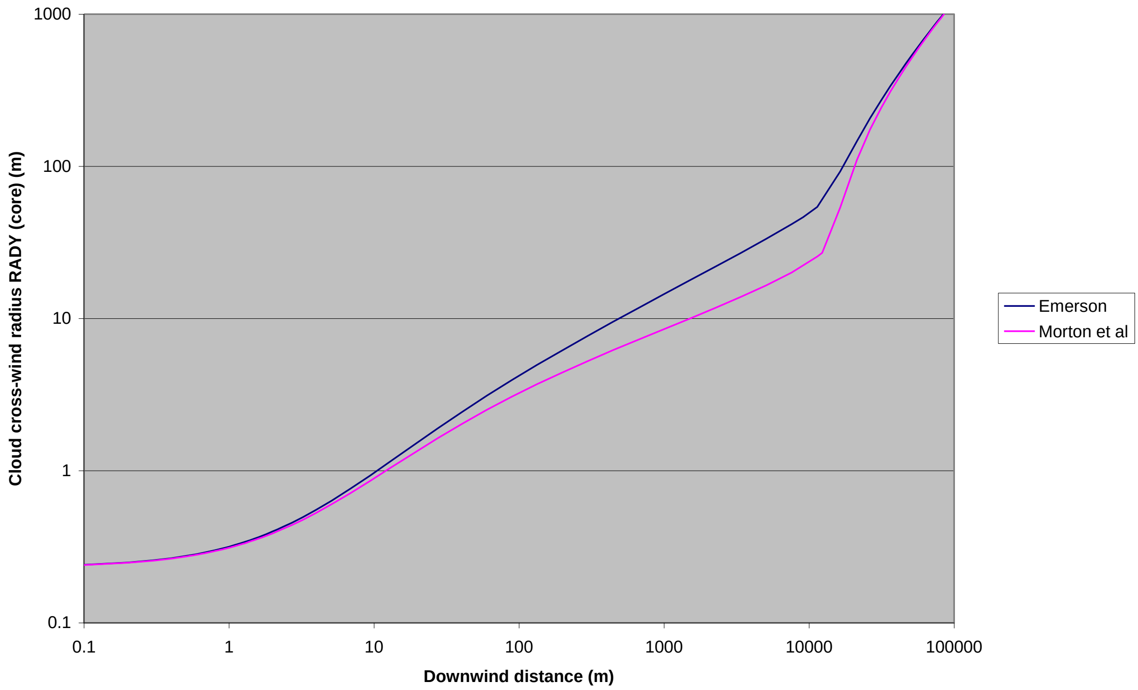
The ratio being larger for materials whose density is greater than that of air and smaller for materials whose density is less than that of air. This relationship is evident in Figure 1(a) where it can be seen close to the source where u_{cld}>>u_a the difference is small. In the far field, however, as the cloud velocity approaches the ambient wind speed the difference becomes much larger.

Centre-line concentration (core)

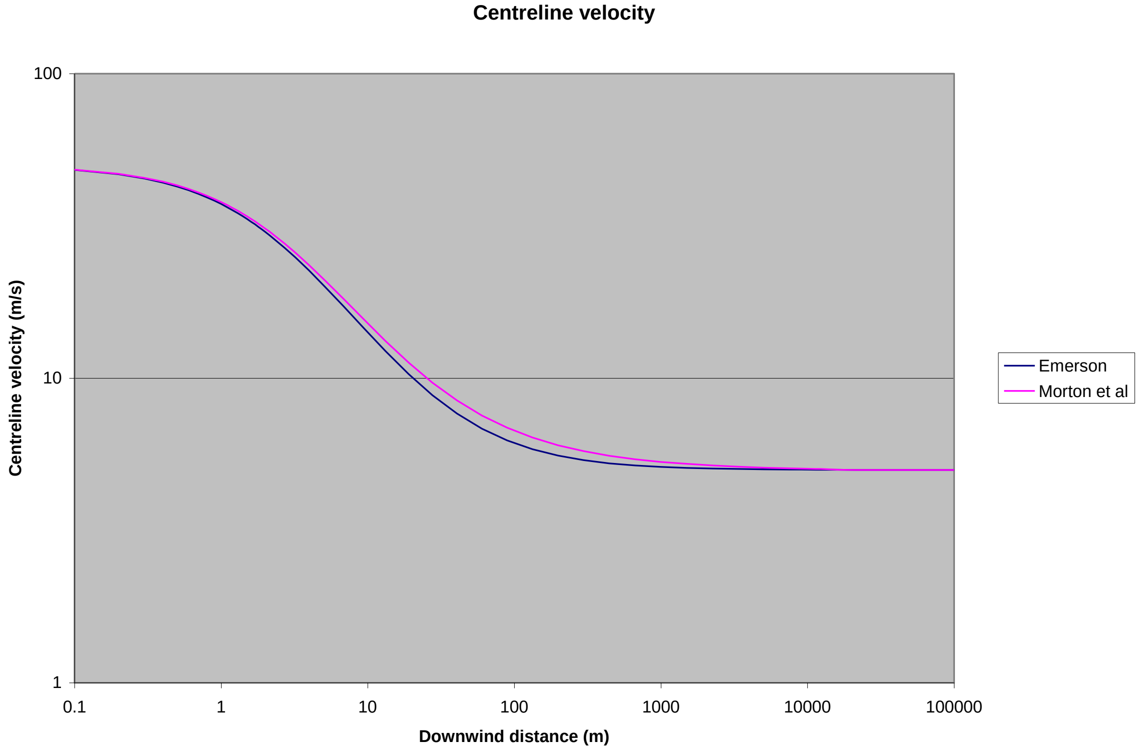


(a) centre-line concentration c_o

Cloud cross-wind radius RADY (core)



(b) cloud radius R_y=R_z



(c) centre-line velocity

Figure 3.1. Comparison of the jet models of Emerson and Morton et al.

3.3 Vertical elevated jet (cross-wind entrainment, air-drag, buoyancy)

In this section the various Phast formulations for the vertical entrainment, cross-wind-entrainment and airborne drag in the UDM model are verified against established plume-rise correlations. Additional work has been done comparing the UDM to a wider range of field scale and wind-tunnel vertical and angled releases. These efforts are described in the UDM Validation manual.

3.3.1 Validation against Pratte and Baines plume-rise correlation (no ambient turbulence)

In this section the formulations for the vertical entrainment, cross-wind-entrainment and airborne drag in the UDM model are validated for a vertical jet in a horizontal cross-flow with a uniform velocity. Entrainment resulting from ambient turbulence is considered not to be present.

Pratte and Baines correlation

The validation of the UDM model is carried out by means of comparison against an empirical correlation by Pratte and Baines[iv] for plume rise,

$$<\Delta z_{cld}> \; = \; <s> \qquad \text{for } 0.1 < <s> < 2.08$$
$$= \; 1.63 <s>^{1/3} \qquad \text{for } 2.08 < <s> < 300$$

Here the dimensionless distance <i> (i = s, or i = Δz_{cld}) is given by

$$< i > = \frac{i}{S \, d_s \sqrt{\rho_{jet} / \rho_a}}$$

with

- Δz_{cld} = plume elevation (m)
- s = distance along the centreline (m)
- S = release velocity / ambient wind speed
- d_s = release diameter (m)
- ρ_a = air density (kg/m^3)
- ρ_{jet} = initial jet density (kg/m^3)

The above correlation was obtained from data for air dispersing into a cross-flow for a range of release diameters and velocity ratios. The experiments were carried out in a wind tunnel, and were characterised by the absence of ambient turbulence.

UDM simulations

The following input data were used as test cases for the UDM in which the transition to passive dispersion was disabled in order to simulate dispersion within a wind tunnel.

- Release material = Pseudo component representing air
- Release direction of 90° with the horizontal
- Stability class D
- Wind speed = 1m/s
- Release height = 1m
- Uniform profiles for temperature, pressure and wind speed

Four UDM runs were carried out as indicated in the table below. The first three cases (R13, R4.6, R43) corresponding to a jet with ambient density (ρ_{jet} = ρ_a). The last case (R33) corresponds to a buoyant methane jet (ρ_{jet} = 0.59 ρ_a). It corresponds to an experiment by Hoehne and Luce[v] and is shown by Post**Error! Bookmark not defined.** to fit well with t he Pratte and Baines correlation.

	release velocity (m/s)	release diameter (m)	release rate (kg/s)	wind speed (m/s)	<s>/s (m^{-1})	<Δz_{cld} >/Δz_{cld} (m^{-1})
POSTBC1	13	0.004	1.93x10^{-4}	1	19.231	19.231
POSTBC2	9.2	0.05	0.02132	2	4.3478	4.3478
POSTBC3	43	0.01	0.003985	1	2.3256	2.3256
POSTBYNT	33	0.0095	0.0016285	1	4.1528	4.1528

Comparison of the UDM predictions were made using Phast 8.4, which includes two new (non-default) methods for modelling crosswind effects in comparison to previous releases. These are described in the UDM Theory Manual, but briefly they are:

- Ooms: Crosswind entrainment has an additional cosθ term as described in the UDM theory manual, with α_2 = 0.43 and C_d = 0.13
- Morton (crosswind modified): Crosswind entrainment is phased in and drag phased out over 'suppression' distances

The simulations below include runs for the default Phast (Morton) and for the two above methods.

UDM results

The results of the comparison are presented in Figure 3.2 for plume rise and in Figure 3.3 for centre-line concentrations (case of R33 only). The conclusions are as follows:

1. In general all models compare well with the Pratte & Baines correlation. Generally the Ooms model underpredicts the Phast 8.4 results, and there is very little difference in plume rise between the default Phast 8.4 and crosswind-modified models.

2. All models predict greater plume rise downwind for the buoyant case than Pratte & Baines (Figure 3.2d). This overprediction is larger in comparison with the case of a non-buoyant plume with similar velocity ratio (Figure 3.2a, Figure 3.2b).

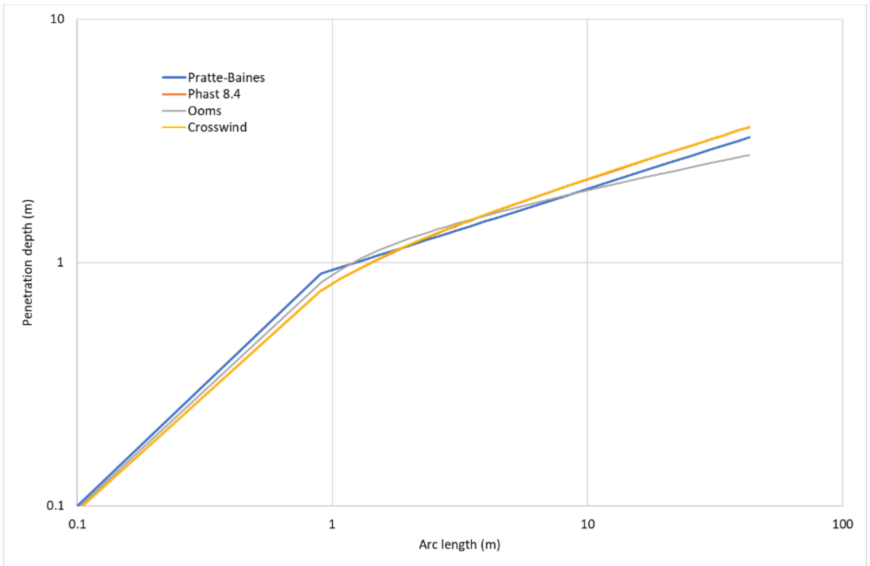
3. Figure 3.3 shows that the centre-line concentrations are significantly underpredicted, though the modified crosswind concentrations are higher, and the Ooms results closest of all[1]. Note that results are scaled up from the dimensionless centre-line mole fraction $<X_j>$ by

$$<X_j> = X_j \; S \; \frac{M_{jet}}{M_a}$$

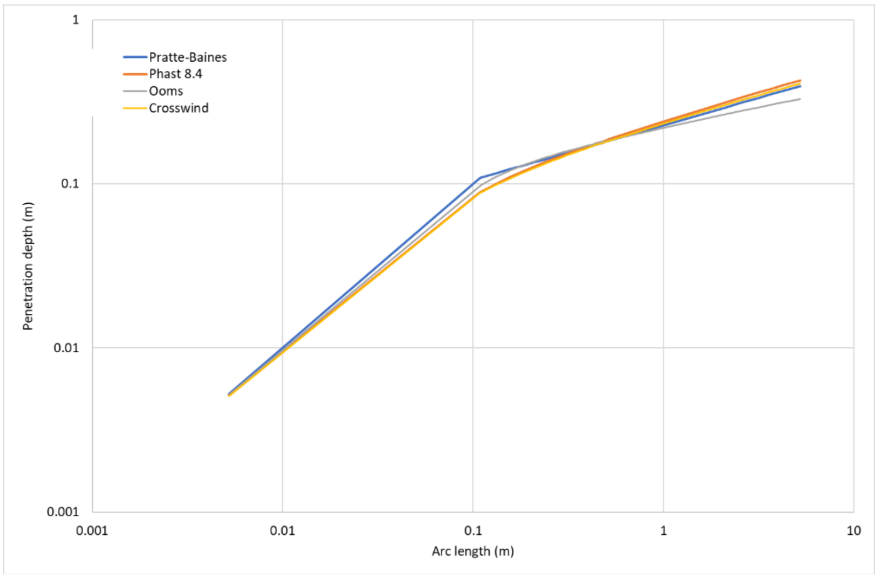
with

- X_j = centre-line mole fraction
- S = release velocity / ambient wind speed
- M_{jet} = molecular weight of release material (methane), kg/kmol
- M_a = air molecular weight , kg/kmol

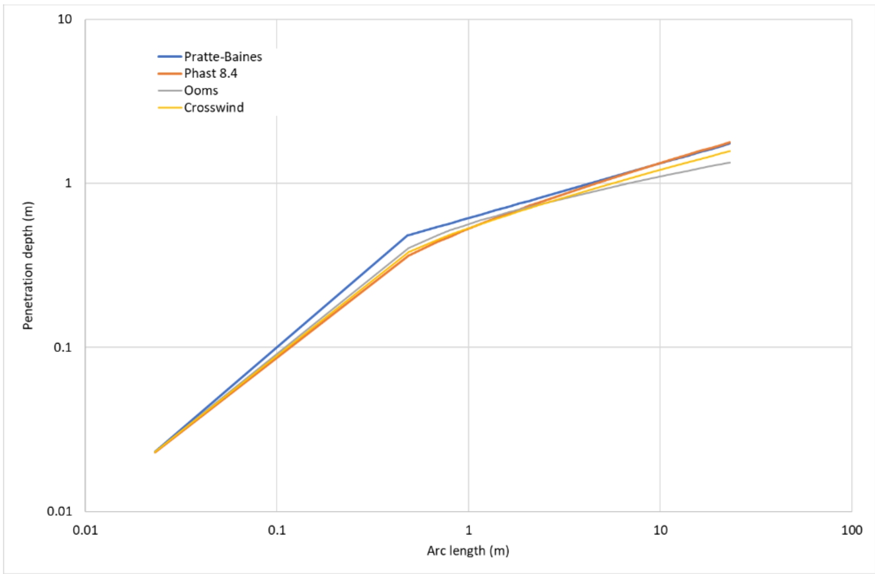
[1] Note this trend is not shown for other vertical or angled experiments, as discussed in the UDM Validation manual.



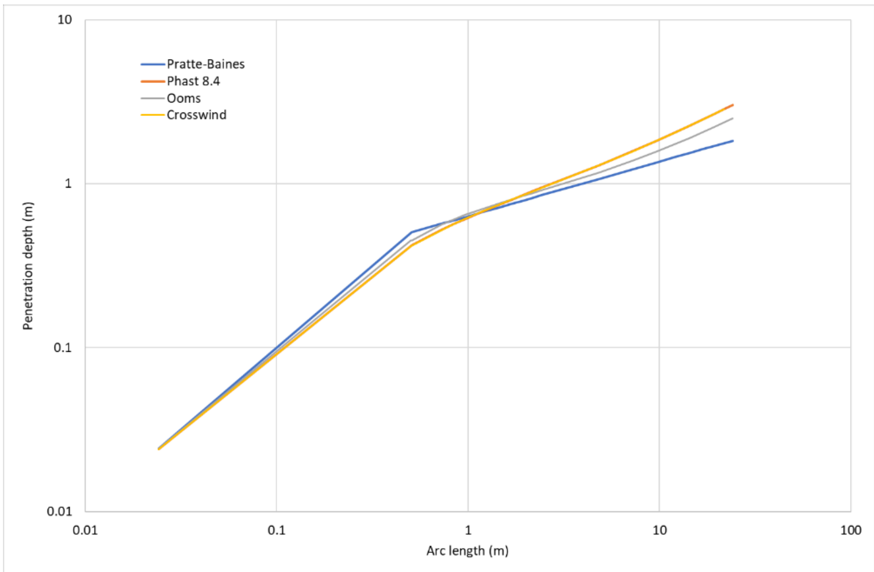
(a) neutrally buoyant plume (velocity ratio = 43)



(b) neutrally buoyant plume (velocity ratio = 13)



(c) neutrally buoyant plume (velocity ratio = 4.6)



(d) - buoyant plume with density ratio 0.59 (velocity ratio = 33)

Figure 3.2. Comparison of UDM results against the correlation of Pratte and Baines Error! Bookmark not defined.

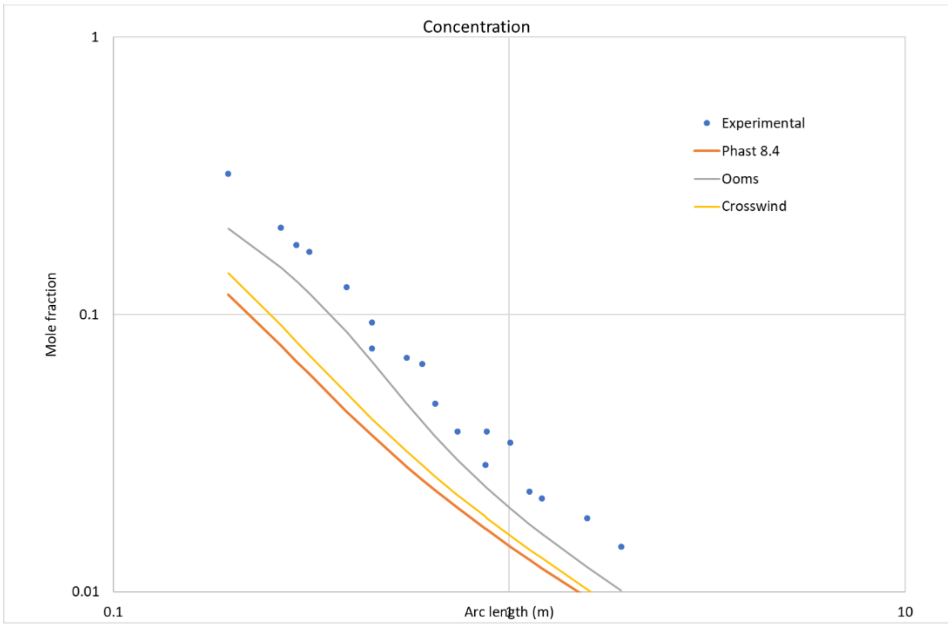


Figure 3.3. Comparison of UDM results (C_D=0) for centre-line concentrations against experimental data by Hoehne and Luce[v]
[buoyant plume with density ratio 0.59 (velocity ratio = 33)]

3.3.2 Validation against Briggs plume rise correlation (with ambient turbulence)

In this section the formulations for the vertical entrainment, cross-wind-entrainment and airborne drag in the UDM model are validated for a vertical jet in a horizontal cross-flow with a uniform velocity. Entrainment resulting from ambient turbulence is present.

Briggs correlation

The validation of the UDM model is carried out by means of comparison against an empirical correlation by Briggs[vi] for plume rise. For non-buoyant releases the correlation is given by the expression[2]:

$$\Delta z_{cld}(x) = \left\{ \frac{3}{4}\left[\frac{d_s\, S^2}{1+0.33S}\right]^2 x \right\}^{\frac{1}{3}} \quad , \quad x < x' = 4d_s\left\{S + 6 + \frac{9}{S}\right\}$$

$$= \Delta z_{cld}(x') \quad , \quad x \geq x'$$

where x is the downwind distance (m).

The downwind distance x' to the maximum plume rise represents the point at which the ambient turbulence is sufficient to dilute the cloud to an extent that negates the effects of vertical momentum. The above equation for x' is taken from the TNO plume-rise formula[vii] adopted in the DNV building wake model BWM[viii,3]. This equation gives similar results to the formula for the maximum plume rise reported by Briggs**Error! Bookmark not defined.**:

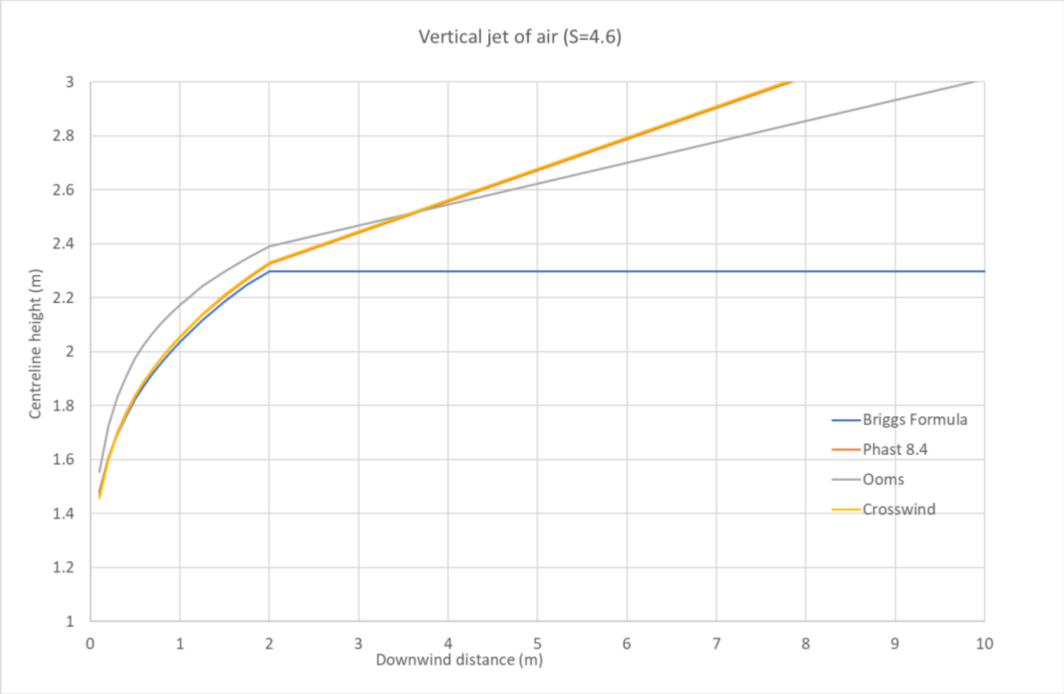
$$\Delta z_{cld,\max} = \frac{4.35\, d_s S}{1 + \frac{3}{S}}$$

UDM results

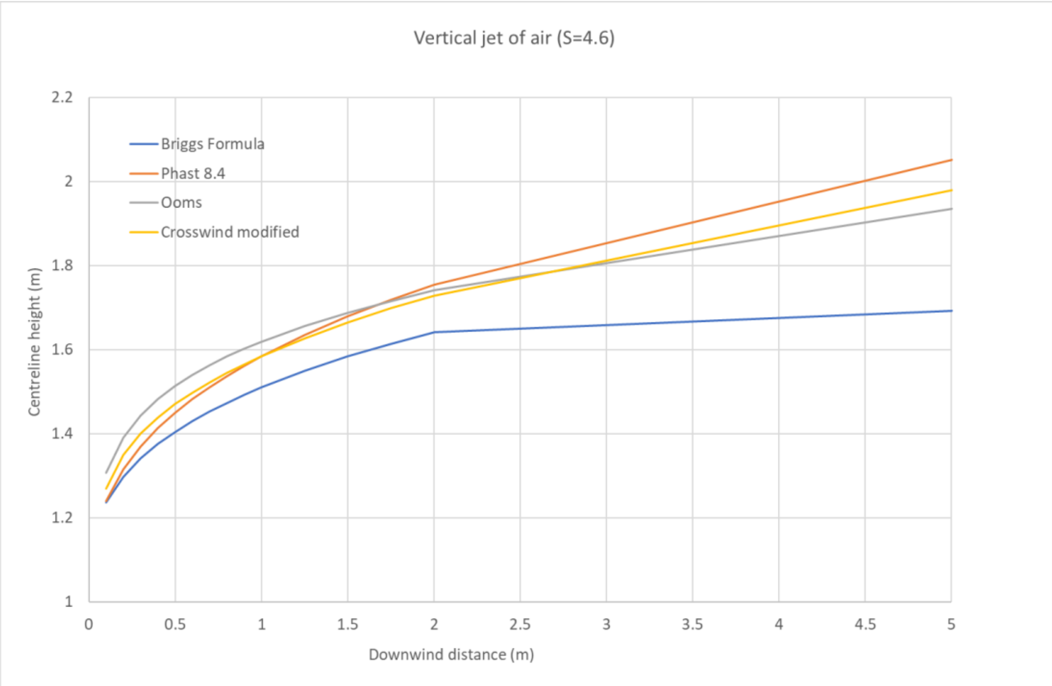
Comparison of the UDM predictions with the plume rise formulae of Briggs is presented in Figure 3.4. For high velocity ratios the trajectory of the default Phast 8.4 and crosswind modified correlations agrees closely with Briggs up until x'. In other circumstances the Ooms model tends to climb faster initially (due to reduced initial entrainment) and flatten off sooner (due to a sustained drag term).

[2] CHECK. Retrieve and check against original reference, e.g. compared to https://www.osti.gov/servlets/purl/5591108/

[3] JUSTIFY. Use of TNO plume rise formula



(a) neutrally buoyant plume (velocity ratio = 43)



(b) neutrally buoyant plume (velocity ratio = 4.6)

Figure 3.4. Comparison against the plume rise formulae of Briggs Error! Bookmark not defined.

3.4 Sensitivity analysis

A limited sensitivity analysis has been carried out for the base case jet release (see section 1.1). Release height, release velocity, release angle and passive transition criteria have been varied. The results[4] are presented in figures 4a – 4d below, from which the following observations can be made.

- Release height. The concentration versus downwind distance graph illustrates some interesting features, namely :

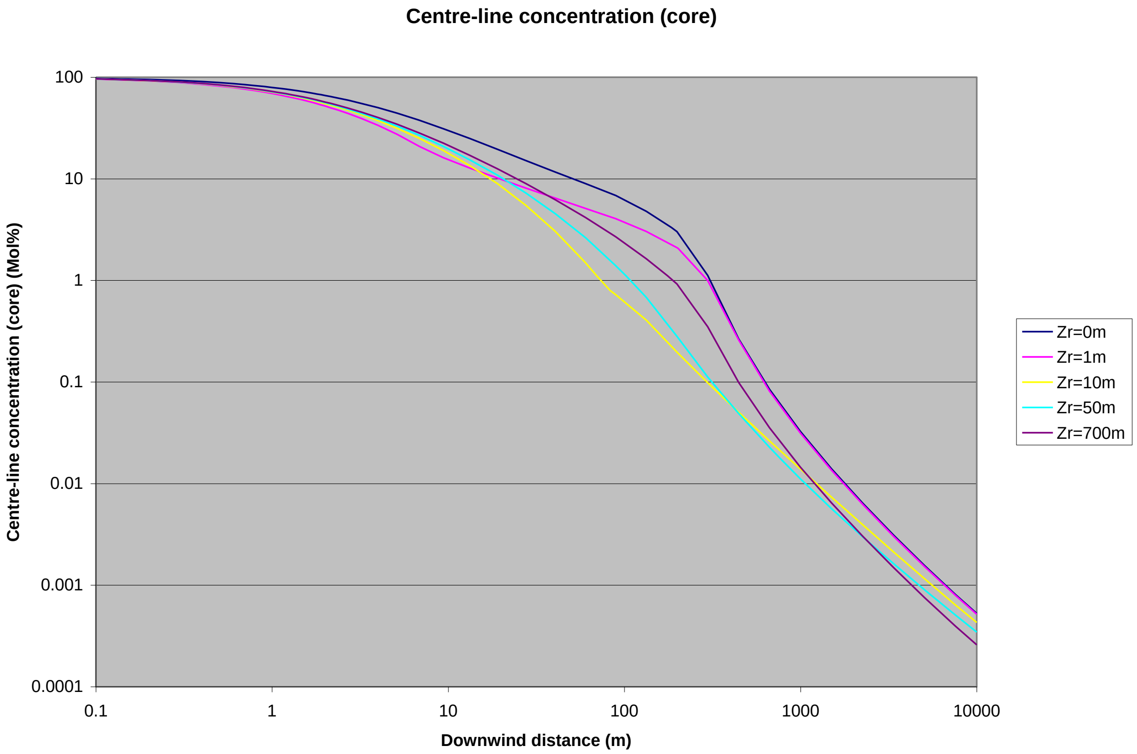
    1. The concentrations for the 10m release are lower than those for the 50 and 700m releases. This is because the near-field passive entrainment decreases with increasing elevation
    2. The concentration for the 10 m release decrease below those for the 1 m release This is because the near field passive entrainment is phased out as the cloud touches down.
    3. The concentrations for the ground level release are much higher than those for the elevated releases. This is because the heavy gas entrainment is much lower than the near field passive entrainment.

- Release velocity. In the near field, the trends in the concentration versus downwind distance are as expected. Further afield the concentrations for the low velocity releases fall below those of the high velocity releases due to the earlier transition to passive entrainment.

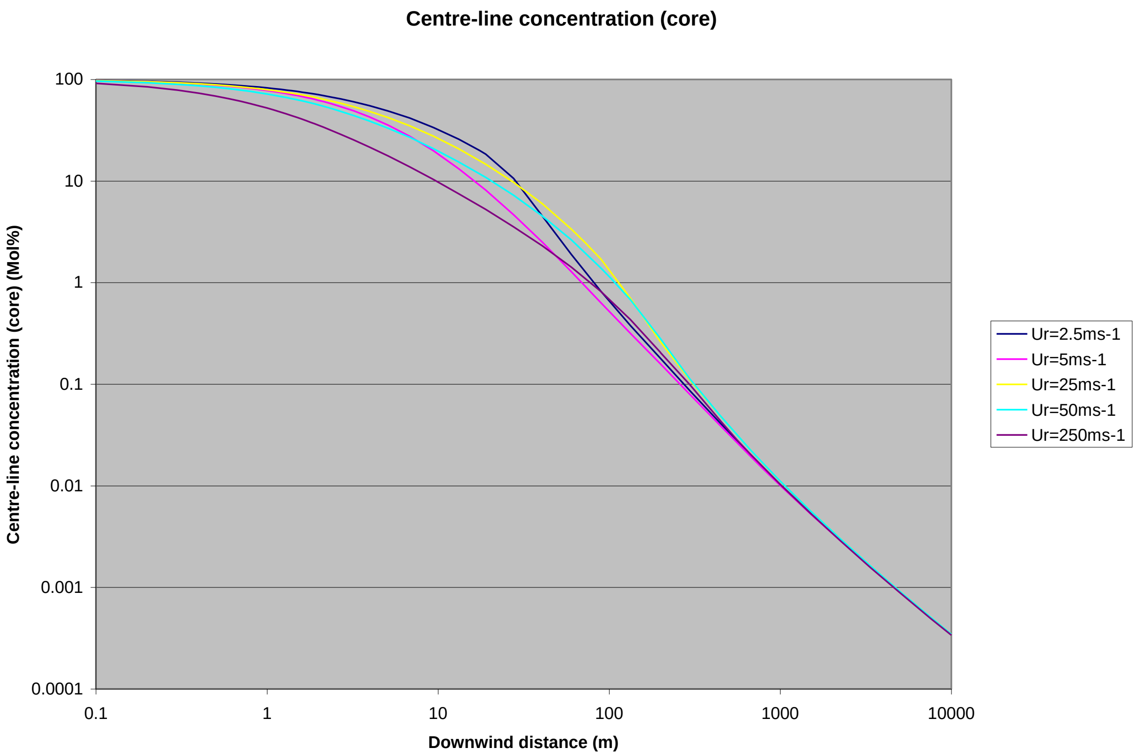
- Release angle. The centreline concentration versus downwind distance graph shows the expected trends, whereby, the larger the release angle the lower the concentration. This is due to the additional crosswind entrainment term and the longer path length for the non-horizontal releases.

- Transition criteria. Varying the velocity transition criteria [$u_{cld}/u_{air} - 1 < r_u$, with r_u = 0.05, 0.1,0.2, 0.3] has little effect upon the results[5].

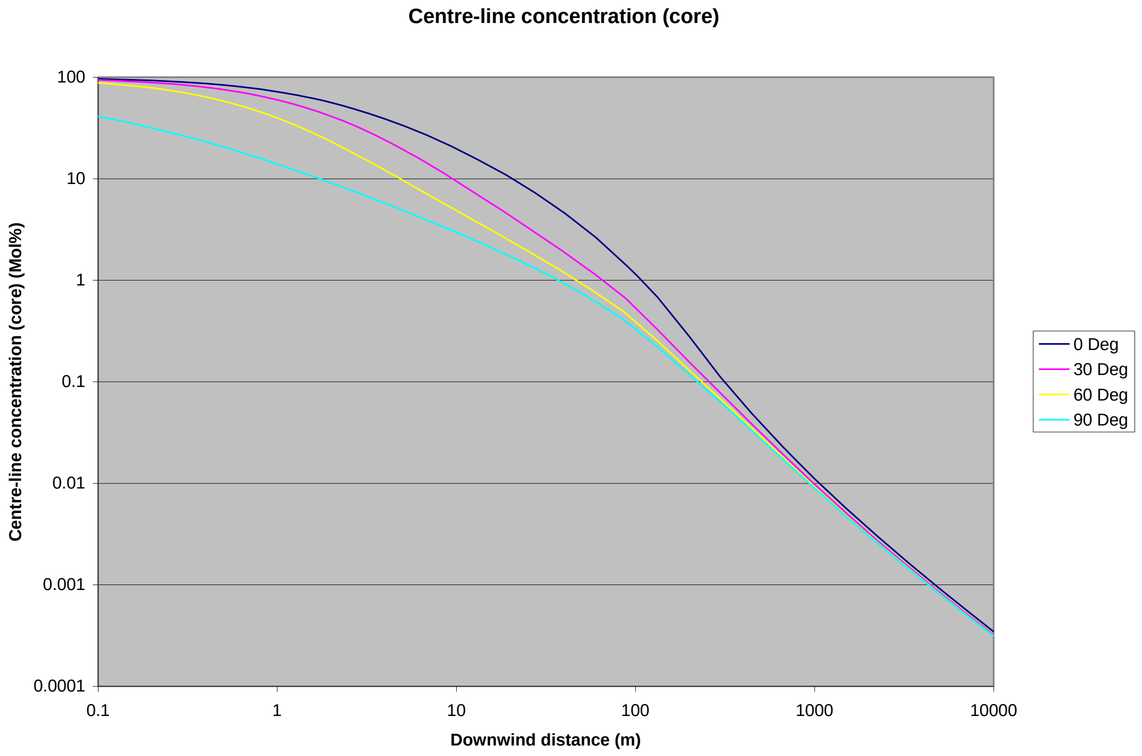
[4] REDO. These figures should be redone using latest version of UDM model
[5] For this specific case, variation of the transition criterion has little effect since the near-field passive dispersion happens to be close the far-field passive dispersion.



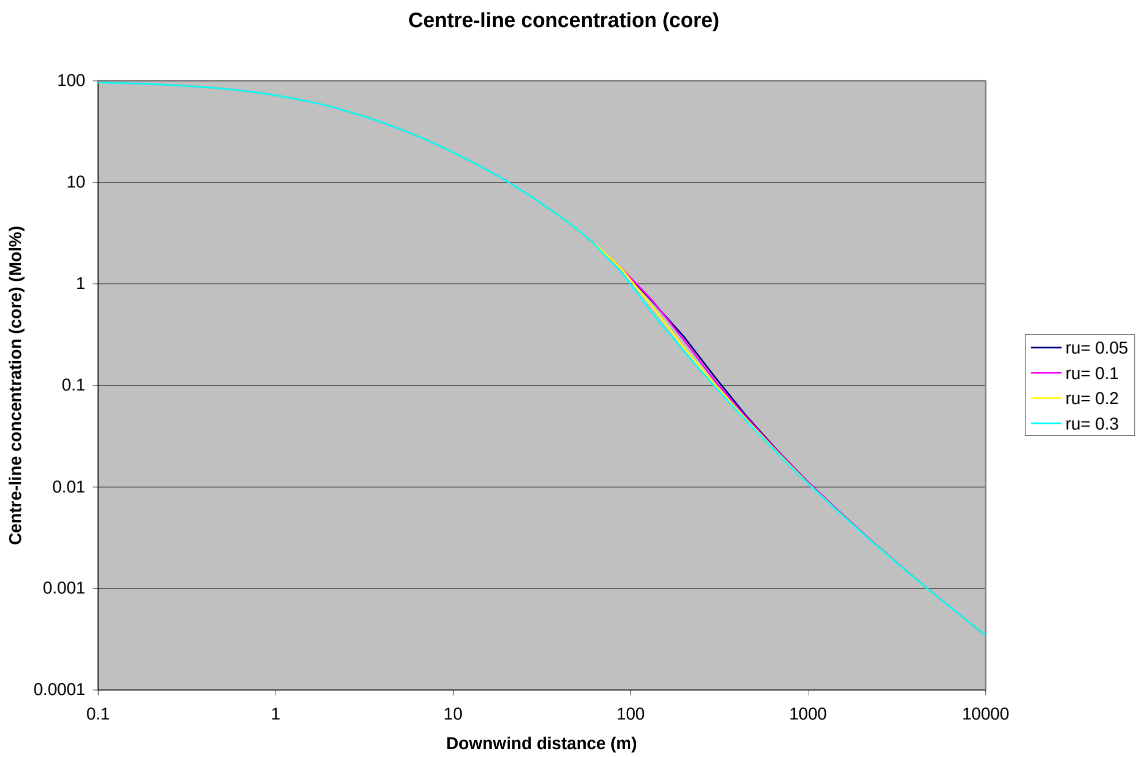
(a) release height



(b) release velocity



(c) release angle with the horizontal



(d) relative cloud velocity transition parameter

Figure 3.5. Centre-line concentration versus downwind distance

3.5 Further work

Possible additional verification and validation may include the following:

- Literature review and validation for ground drag force
- Literature review, updated modelling and validation for plumes interacting with the ground
    - Ground drag force
    - ‘Attachment’ for high velocity plumes close to the ground
    - Ground impact force

- Additional sensitivity analysis for jet dispersion in a cross flow

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